Proof of the Pythagoras theorem.

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2 Proof of the Pythagorean theorem



Introduction



Introduction objective

In this presentation we try to show a proof of the Pythagorean theorem. There are many demonstrations, but this one is one of the simplest.



Concept

Suppose we have a square of side \mathbf{r} and on each of its sides we place a right triangle of legs \mathbf{x} and \mathbf{y} . As in this situation the hypotenuse of each of the triangles is \mathbf{r} we want to prove that:

Formula

$$x^2 + y^2 = r^2$$

Proof of the Pythagorean theorem



The figure

The figure that is obtained is the following:





Conclusions

Each side of the green square is the sum of x and y. Therefore, the area of the square is:

For the same reason, the area of the red square is:

The area of each of the green triangles (y, x and r) is:

$$\frac{x+y}{2}$$

 $(x + y)^2$

 r^2



Demonstration

The green square is formed by the red square and the four green triangles, so the sum of all the areas is:

$$(x+y)^2 = r^2 + 4(\frac{x+y}{2})$$

We develop the left part of equality:

$$(x+y)^2 = x^2 + 2xy + y^2$$

We substitute in the first formula:

$$x^2 + 2xy + y^2 = r^2 + 2xy$$

2xy is eliminated on both sides of the equality, and we obtain the desired result:

$$x^2 + y^2 = r^2$$