Part 3: k-nearest neighbours

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k-nearest neighbours algorithm

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In a basic version:

- represent all your training samples in n-dimensional space,
- define a distance function between points (for example Euclidean distance),
- choose $k \in \mathbb{N}$,
- for a given observation in the test set find k observations (neighbours) from the train set which distance from your observation is the smallest,
- predict value/class of your observation based on values/classes of your neighbours.

The algorithm is widely used for classification and regression purposes. In the simplest form there is no parameters and only one hiperparameter - number of neighbours.

KNN in recommender systems

User-based KNN in recommender systems

Suppose we have m users and n items and we want to predict a rating r_{ui}.

- each **user** is represented by his ratings,
- we define a **similarity measure** $\mathrm{sim}(u, v)$ **between users** - it is common to take cosine similarity or Pearson coefficient of the vectors ratings of items rated by both users,
- we choose $k \in \mathbb{N}$,
- we are looking for a set $N_i^k(u)$ of k **the most similar users** to the user u who rated movie i
- the final prediction is:

$$\hat{r}_{ui} = \frac{\sum_{v \in N_i^k(u)} \mathrm{sim}(u, v) r_{vi}}{\sum_{v \in N_i^k(u)} \mathrm{sim}(u, v)}.$$

KNN in recommender systems

Item-based KNN in recommender systems

Suppose we have m users and n items and we want to predict a rating r_{ui}.

- each **item** is represented by received ratings,
- we define a **similarity measure** $\mathrm{sim}(i, j)$ **between items** - it is common to take cosine similarity or Pearson coefficient of the vectors ratings of items restricted to users who rated both items,
- we choose $k \in \mathbb{N}$,
- we are looking for a set $N_u^k(i)$ of k **the most similar items** to the item i which were rated by user u,
- the final prediction is:

$$\hat{r}_{ui} = \frac{\sum_{j \in N_u^k(i)} \mathrm{sim}(i, j) r_{uj}}{\sum_{j \in N_u^k(i)} \mathrm{sim}(i, j)}.$$

Difference between user-based KNN and item-based KNN

User/Item	i_1	i_2	i_3	i_4	i_5
u_1	1	0	1	1	0
u_2	0	1		1	0
u_3		1	0	1	0
u_4		1	1	1	1
u_5	1	0	1	?	?

Figure: User-item rating matrix

User-based approach:

$$s(u_5, u_1) = \cos([1,0,1],[1,0,1]) = 1$$
$$s(u_5, u_2) = \cos([0,1],[1,0]) = 0$$
$$s(u_5, u_3) = 0$$
$$s(u_5, u_4) = \frac{1}{\sqrt{2}}$$
$$\hat{r}_{u_5,i_4} = r_{u_1,i_4} = 1$$
$$\hat{r}_{u_5,i_5} = r_{u_1,i_5} = 0$$

Item-based approach:

$$s(i_4, i_1) = \cos([1,1],[0,1]) = \frac{1}{\sqrt{2}} \approx 0.71$$
$$s(i_4, i_2) = \frac{3}{2\sqrt{3}} \approx 0.87$$
$$s(i_4, i_3) = \frac{2}{\sqrt{6}} \approx 0.82$$
$$\ldots$$
$$\hat{r}_{u_5,i_4} = r_{u_5,i_2} = 0$$
$$\hat{r}_{u_5,i_5} = r_{u_5,i_3} = 1$$

Some KNN problems

KNN problem - lack of similar users/items

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In practice number of users and items might be huge (several millions) and preserving similarity matrix in memory is expensive. Unfortunately preserving only top k similar users/items for each item is not enough (check carefully formulas to answer why).

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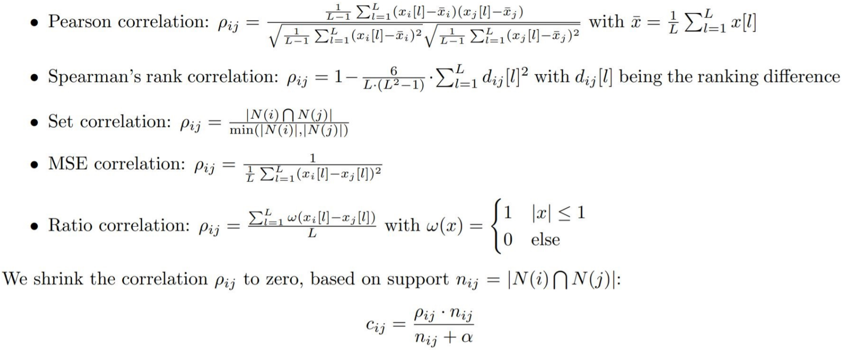
Solution

Preserve only the most similar users. If they have not rated target movie, use the prediction of their ratings instead. In [1] MF model was used for predictions.

Similarity measures



- Pearson correlation: $\rho_{ij} = \frac{\frac{1}{L-1}\sum_{l=1}^{L}(x_i[l]-\bar{x}_i)(x_j[l]-\bar{x}_j)}{\sqrt{\frac{1}{L-1}\sum_{l=1}^{L}(x_i[l]-\bar{x}_i)^2}\sqrt{\frac{1}{L-1}\sum_{l=1}^{L}(x_j[l]-\bar{x}_j)^2}}$ with $\bar{x} = \frac{1}{L}\sum_{l=1}^{L} x[l]$
- Spearman's rank correlation: $\rho_{ij} = 1 - \frac{6}{L\cdot(L^2-1)}\cdot\sum_{l=1}^{L} d_{ij}[l]^2$ with $d_{ij}[l]$ being the ranking difference
- Set correlation: $\rho_{ij} = \frac{|N(i)\bigcap N(j)|}{\min(|N(i)|,|N(j)|)}$
- MSE correlation: $\rho_{ij} = \frac{1}{\frac{1}{L}\sum_{l=1}^{L}(x_i[l]-x_j[l])^2}$
- Ratio correlation: $\rho_{ij} = \frac{\sum_{l=1}^{L}\omega(x_i[l]-x_j[l])}{L}$ with $\omega(x) = \begin{cases} 1 & |x| \le 1 \\ 0 & \text{else} \end{cases}$

We shrink the correlation ρ_{ij} to zero, based on support $n_{ij} = |N(i)\bigcap N(j)|$:

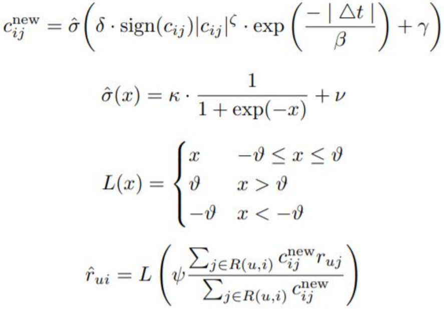
$$c_{ij} = \frac{\rho_{ij}\cdot n_{ij}}{n_{ij}+\alpha}$$

Similarity measures used by the winners of Netflix Prize [2]

KNNMovieV3-2

Models used by the winners of the Netflix Prize competition

Here is an example of k-NN model used in 33 predictors (25 predictors with simpler approach ($\zeta = \kappa = \psi = 1, \nu = 0, \vartheta = \infty$)).



$$c_{ij}^{\text{new}} = \hat{\sigma}\left(\delta \cdot \text{sign}(c_{ij})|c_{ij}|^{\zeta} \cdot \exp\left(\frac{-\mid \Delta t \mid}{\beta}\right) + \gamma\right)$$

$$\hat{\sigma}(x) = \kappa \cdot \frac{1}{1 + \exp(-x)} + \nu$$

$$L(x) = \begin{cases} x & -\vartheta \leq x \leq \vartheta \\ \vartheta & x > \vartheta \\ -\vartheta & x < -\vartheta \end{cases}$$

$$\hat{r}_{ui} = L\left(\psi \frac{\sum_{j \in R(u,i)} c_{ij}^{\text{new}} r_{uj}}{\sum_{j \in R(u,i)} c_{ij}^{\text{new}}}\right)$$

KNN models used by the winners of Netflix Prize [2]

Python implementation

To do (especially for absent students):

- Go through - *P3. k-nearest neighbours* notebook to:
  - check simplified version of I-KNN (where we sum over all neighbours instead of top k)
  - observe evaluation measures
  - run ready-made KNN algorithm implemented in Surprise
  - read Surprise docs about KNN algorithms here, it is described really clear
  - **project task 4: use a version of your choice of Surprise KNN algorithm**

References I

[1] A. Töscher, M. Jahrer, and R. Legenstein, “Improved neighborhood-based algorithms for large-scale recommender systems,”, Jan. 2008. DOI: 10.1145/1722149.1722153.

[2] A. Töscher and M. Jahrer, “The bigchaos solution to the netflix grand prize,”, Sep. 2009, http://https://www.netflixprize.com/assets/GrandPrize2009_BPC_BigChaos.pdf/.