

Part 5: Graph-based models

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Different types of graph models

There exists many recommendation models utilizing graphs.

Depending of what is the node in the graph we distinguish three basic categories:

- user-user graphs where nodes are users
- item-item graphs where nodes are items
- user-item graphs where nodes are both users and items

It is also possible to treat other objects as nodes - for example some features of items or users like tags or categories.

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- a prediction of a rating $\hat{r}_{ui}^{(v)}$ is a rating of r_{vi} under the composition of linear transformations of edges on the shortest path connecting u and v ,
- a final prediction of a rating \hat{r}_{ui} is an average of $\hat{r}_{ui}^{(v)}$ over distinct v 's

Item-item graphs

Item-item graph is an alternative way of defining similarity measure over items and might be used by other algorithms (for example kNN).

	GLADIATOR	GODFATHER	BEN-HUR	GOODFELLAS
U_1	1			5
U_2		5		
U_3	5	3		1
U_4			3	
U_5				3
U_6	5		4	

(a) Ratings matrix



(b) Unnormalized correlation graph (c) Normalized correlation graph

Figure: Item-item graph. Source: [1]

User-item bipartite graph

In user-item graphs:

- each user and item is represented by a node,
- the edge between nodes exists if there was an interaction between given user and item,
- it is possible to add weights between edges.

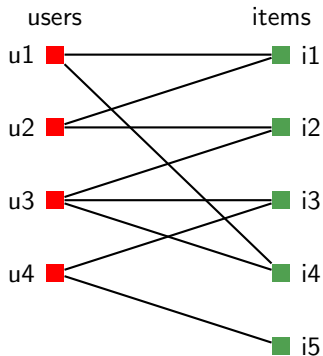


Figure: Bipartite graph of 5 users and 5 items

Adjacency matrix

Adjacency matrix of a graph of n nodes is a $n \times n$ matrix where $a_{i,j}$ is the number of edges connecting nodes i and j .

In our example we have

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Note that $A^s(i,j)$ is the number of paths connecting vertices i and j in exactly s steps.

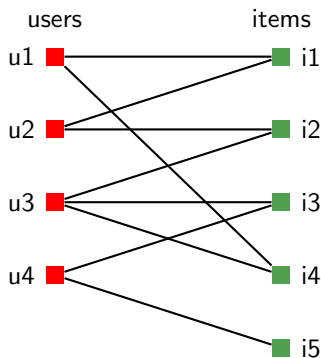


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3-Paths model

For a given user u we can recommend items with the greatest number of paths of a given order starting at node u .

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Possible solution

Instead of using adjacency matrix we can use transition matrix (our process here is a Markov chain).

To receive transition matrix it is enough to divide each row by its sum.

P3 model

For a given user u we can recommend items with the greatest probability of transition from u within exactly 3 steps.

For convenience we will consider user and item transition matrices. In our case we will have

$$P_{ui} = \begin{bmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

$$P_{iu} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note that for the matrix $P3 = P_{ui} \cdot P_{iu} \cdot P_{ui}$, $P3(u, i)$ contains a probability that a random walk of the order 3 starting from u will finish at i .

In [2] Cooper et al. introduced a simple modification of P3 model. Basically they have taken each entry of a transition matrix to the power α , where α is a hyperparameter.

P3alpha model

For a given user u we can recommend items ordered by $P3\alpha = (P_{ui})^\alpha \cdot (P_{iu})^\alpha \cdot (P_{ui})^\alpha$, where $(P_{ui})^\alpha$ is a matrix (P_{ui}) taken elementwise to the power α .

Note that:

- $P3\alpha$ is no longer a transition matrix,
- with increasing α we decrease the impact of popular nodes,
- P3 is a special case of $P3\alpha$ for $\alpha = 1$.

In [3] Paudel et al. introduced another simple improvement of P3 (and of P3alpha).

RP3beta model

For a given user u we can recommend items ordered by $P3alpha$ divided by item popularity to the power $beta$.

Note that:

- $RP3beta$ reduces to $P3alpha$ for $\beta = 0$,
- with increasing $beta$ we recommend popular items less often.

Random walks

Matrix multiplication in case of large datasets is not possible.
Several ways ([2], [3]) introduced effective way of estimating graph based approaches by using random walk sampling.

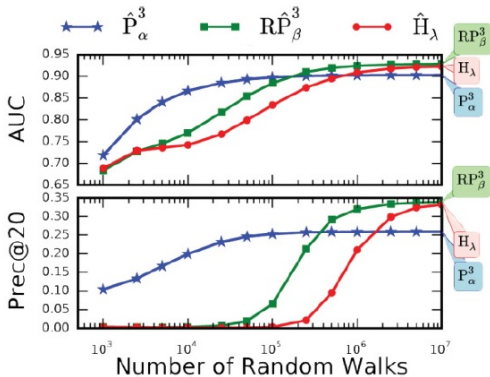


Figure: AUC and Prec@20 for different number of random walks per user generated on MovieLens-M dataset [3]

To do (especially for absent students):

- Go through - *P5. Graph-based* notebook to:
 - go through the implementation of RP3Beta
 - save recommendations of P3 model
 - observe evaluation measures
 - optimize hiperparameters
 - **project task 6: generate recommendations of RP3Beta for hiperparameters found to optimize recall**
 - **project task 7 (optional): implement graph-based model of your choice (for example change length of paths in RP3beta)**
 - observe sample recommendations

Project tasks

- project task 1: implement TopRated
- project task 2: implement self-made BaselineU
- project task 3: implement some other evaluation measure
- project task 4: use a version of your choice of Surprise KNN algorithm
- project task 5: implement SVD on top baseline (as it is in Surprise library)
- project task 6: generate recommendations of RP3Beta for hiperparameters found to optimize recall
- project task 7 (optional): implement graph-based model of your choice (for example change length of paths in RP3beta)

References I

- [1] C. Aggarwal, “Recommender systems,” Jan. 2016, <https://link.springer.com/book/10.1007/978-3-319-29659-3>.
- [2] C. Cooper, S. H. Lee, T. Radzik, and Y. Siantos, “Random walks in recommender systems: Exact computation and simulations,” *WWW '14 Companion*, pp. 811–816, 2014.
- [3] B. Paudel, F. Christoffel, C. Newell, and A. Bernstein, “Updatable, accurate, diverse, and scalable recommendations for interactive applications,” *ACM Transactions on Interactive Intelligent Systems*, vol. 7, pp. 1–34, Dec. 2016.